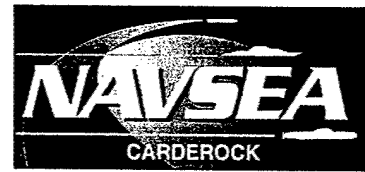


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NSWCCD-50-TR-2003/015 March 2003

Hydromechanics Directorate Report

## A Proposed Criterion for Launch Ramp Availability

By  
J.F. Dalzell

NSWCCD-50-TR-2003/015 A Proposed Criterion For Launch Ramp Availability



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## ABSTRACT

*The project under which the present report was produced has as an objective the development of methods for the evaluation and comparison of stern-launch and side-launch systems for small boat deployment from USCG cutters. A major difference in the two systems is the presence of a stern launch and retrieval ramp. Under some sea and operational conditions the available water depth over the outboard end of a stern ramp may become limiting to the operation. The objective of the work to be summarized was the development of new criteria for the evaluation of the ramp availability to launch and retrieval operation.*

## ADMINISTRATIVE INFORMATION

This project was funded by the US Coast Guard Engineering Logistics Center under Military Interdepartmental Purchase Request Numbers DTCTG40-01-X-40436 and DTCTG40-03-X-60208. The work was performed by the Carderock Division, Naval Surface Warfare Center Seakeeping Department under Work Unit Numbers 01-1-5500-313 and 03-1-5500-705 with DPI of 5015. This work was completed by the late John F. Dalzell under Purchase Order Number N0016701M0307 to the Carderock Division.

## INTRODUCTION

The overall objective of the project under which the present report was produced was to develop methods for the evaluation and comparison of stern-launch and side-launch systems for small boat deployment from USCG cutters.

The stern launch system involves a ramp of approximately 20° inclination built into the fantail of the cutter. To launch, the boat is lowered part way down the ramp and held in position by a quick release hook while the motors are started. At an appropriate time according to the judgment of the launch personnel, the hook is released and the boat slides down and out of the ramp. If relative motions between the ship and local water surface are large, the judgment about when to release must involve in part the amount of water over the outboard end (the sill) of the ramp—presumably, an unfortunate start might result in the boat hanging up temporarily on the sill.

In the retrieval operation, the boat approaches and lines up on the ramp, and, according to the judgment of the coxswain, accelerates and drives as far up the ramp as

possible (typically about half-way). At this time the winch cable is hooked on and the boat is winched the rest of the way up the ramp. Certainly, it will be un-wise to hit the ramp just as the sill emerges, in fact, there may be a minimum depth of water over the sill for a reliable landing.

Though both require human judgment to minimize adverse effects of ship motion, the retrieval operation clearly appears to be more difficult than the launch.

### INITIAL ANALYSIS

It is assumed that the mission of the small boat is almost always carried out in a relatively short time, and accordingly, that sea conditions are apt to be the same for launch and retrieval. Thus, for any given operation it should suffice to base ramp criteria on the more difficult of the two parts of the mission—the retrieval.

It appears reasonable to assume that under severe conditions the coxswain will watch the ship motion relative to the local water surface and start the drive to the ramp when it appears that the ramp is on the down part of its cycle where the water depth over the sill is greatest. The coxswain needs time to decide to go, to accelerate the boat, and to move the boat onto the ramp. During this time the water depth over the ramp sill needs to be deep enough to as not to contact the bottom of the boat, at least until the bow has “grounded” on the ramp and has been hooked on.

Thus it appeared reasonable to search for a criterion which involves the statistics of the duration that the depth of water over the ramp sill is greater than some specified depth. Such statistics would define a “launch ramp availability” time. The associated criterion would be the minimum time required for the retrieval operation.

The contemplated overall evaluation and comparison approaches involve the conventional linear-random model of the sea and the ship motions. Accordingly, the motion processes are assumed to be Gaussian and zero mean, and these assumptions allow the application of some existing results from threshold crossing theory.

First, the relative vertical motion at the location of the ramp sill,  $Y(t)$ , say, is assumed to be a random Gaussian zero-mean process. Relative vertical motion is defined as the difference between the absolute ship motion at some point on the static waterline and the local water elevation. (It should be noted that some special ship motion computations beyond the usual strip methods may be required to refine the estimates of

the local water elevation just aft of the ship.) Relative motion is zero when there are no waves to excite the ship, and under these circumstances the depth of water over the sill is the nominal still water value.

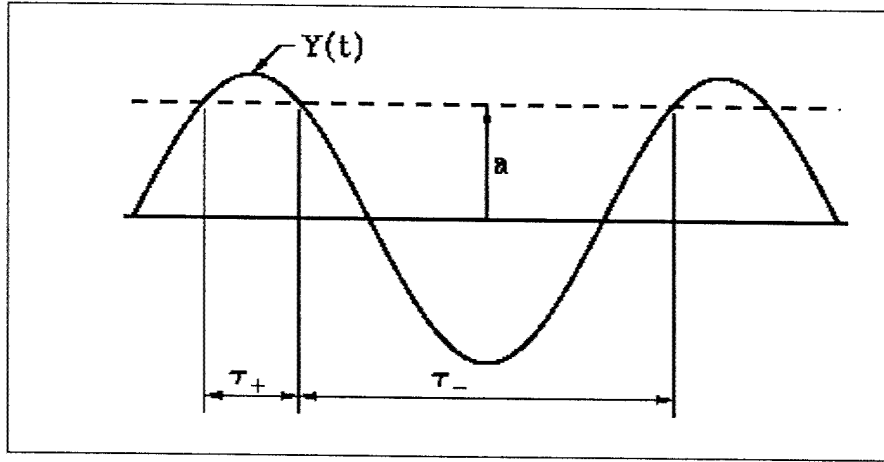


Figure 1: Definition Sketch

Figure 1 sketches a fragment of the relative motion process,  $Y(t)$ , and indicates some definitions which relate the present problem to the mathematical threshold crossing problem. The relative motion process may be defined to be positive when the selected point on the ship rises above the local water level. Thus, if the relative motion is positive and equal to  $a$ , say, the water depth over the sill is the original depth less  $a$ . The value  $a$  is a constant “threshold” that depends upon the geometry of the problem. If  $a$  is set equal to the depth of water over the sill for no-wave conditions, during the time the relative motion,  $Y(t)$ , exceeds  $a$ , the sill emerges. Alternately, if  $a$  is set equal to the nominal depth of water over the sill less an allowance for the draft of the boat, during the time that  $Y(t)$  exceeds  $a$  the water over the sill will be less than the draft of the boat.

Noting Fig. 1, the time duration that the relative motion exceeds the threshold,  $a$ , may be denoted  $\tau_+$ . Similarly, the time duration that the relative motion is less than the threshold may be denoted  $\tau_-$ . Thus the threshold problem of most current interest involves the statistics of  $\tau$  for a specified threshold,  $a$  and random process,  $Y(t)$ .

#### PROPOSED CRITERION FOR RAMP AVAILABILITY

Appendix A contains a review of results and approximations from threshold crossing theory. Appendix B contains a demonstration via simulated data of how well some of these results work.

The average length of the time intervals during which  $Y(t) < a$  will be denoted by  $\langle \tau_- \rangle$ . The theory yields a solid estimator for this quantity as:

$$\langle \tau_- \rangle = \frac{2\pi\sigma}{\sigma_v} \exp\left[(a/\sigma)^2 / 2\right] \Phi(a/\sigma) = T_z \exp\left[(a/\sigma)^2 / 2\right] \Phi(a/\sigma) \quad (1)$$

where  $\sigma$  is the standard deviation of the relative motion process,  $Y(t)$ ,  $\sigma_v$  is the standard deviation of the first derivative of the relative motion process, and  $\Phi(\cdot)$  is the standardized Normal probability integral. Normally, the standard deviations would be computed as the square roots of the zero<sup>th</sup> and second moments of the relative motion spectrum, ( $m_0$  and  $m_2$ ).

It is noted in the alternative forms of Eq. (1) that functions of  $a/\sigma$  modify the mean zero crossing period,  $T_z = 2\pi\sigma/\sigma_v$ , to result in the average time interval. The quantity  $T_z \exp[(a/\sigma)^2 / 2]$  is an estimate for the mean time between threshold up- or down-crossings. What the probability integral does is to determine the part of the average up-crossing period where the process is expected to be below the threshold. When the threshold is zero,  $\Phi(a/\sigma) = 0.5$  and the average time interval during which  $Y(t) < 0$  is half the zero crossing period. When the threshold is very large  $\Phi(a/\sigma)$  approaches unity and the average time interval during which  $Y(t) < a$  approaches the mean time between threshold up-crossings.

The reciprocal of the estimate for the mean time between threshold up- or down-crossings ( $T_z \exp[(a/\sigma)^2 / 2]$ ) is the expected number of threshold crossings per unit time, a computation that has been done in support of deck wetness criteria for many years. The only moderately new thing computationally in Eq. (1) is the probability integral,  $\Phi(\cdot)$ . Fortunately, this semi-infinite integral does not have to be evaluated each time because there are available a number of very accurate and very fast numerical approximations.

The proposed criterion is that ramp operations be considered unsuccessful or more than usually dangerous for a given sea and operational condition if

$$\langle \tau_- \rangle < T_{RA}$$

where  $T_{RA}$  is a minimum average time interval for the retrieval operation.

## INITIAL THOUGHTS ON THE MINIMUM AVERAGE TIME INTERVAL

Like other prescribed values for seakeeping assessment, the selection of a specific value for a minimum average time interval,  $T_{RA}$ , should, in principle, be established in part by feedback from actual operations. At present, we do not have such data, and initial values must be established by educated guess work.

The information available suggests that the entire evolution, from boat arrival in the vicinity of the ship to the completion of securing the boat on deck can take something like one or two minutes. The winching up the ramp after hooking on and the final securing is apt to take a fair fraction of the total. The actual time it may take to move into and ground on the ramp after the boat is up to speed depends on the relative speeds and the starting standoff. If it is supposed that the relative closing speed is three knots, the boat is seven meters in length and the starting standoff where the boat is up to speed is about two boat lengths, the time to ground on the ramp is something less than 10 seconds. If it takes another 10 seconds to make the decision to start and accelerate to the closing speed, the required ramp availability window could be of the order of 20 seconds. A closer starting standoff would of course cut the time as much as a half so that 20 seconds can easily be rather conservative.

On the other hand, under extreme conditions the coxswain probably has to wait for a lull and make a prediction of sorts. The results in the appendices indicates that the distribution of the time intervals of interest is approximately Exponential for ratios of threshold to  $\sigma$  of 1.5 and higher. The assumption of exponentiality means that 63% of the intervals will be *smaller* than the average interval, so that there is a serious chance that the single interval the coxswain chooses will end up being significantly less than the average—more thinking about this risk is warranted.

## ACKNOWLEDGEMENTS

Mr. John Dalzell was an employee of the Naval Surface Warfare Center, Carderock Division from 1984 to 1997 then worked as a contractor to NSWCCD until his death in 2001. He wrote this paper as a technical memorandum in 2001 under contract to NSWCCD a few months before he died. The technical memorandum was converted to a Hydromechanics Directorate report so that the technical contents of the memorandum could be referenced. This was his final technical work.

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## APPENDIX A. RESULTS AND APPROXIMATIONS FROM THEORY

### INTRODUCTION

By and large, the literature has been concerned with the statistics of the durations that a process *exceeds* some threshold (the  $\tau_+$  of Fig. 1), not the durations that the process is below the threshold needed here (the  $\tau_-$  of Fig. 1). For this reason some re-examination of the basic references has been required.

In addition, the final application requires a certain amount of computational simplicity. Graham et al (1991) faced the same problem, but required the statistics of exceedance durations. Though the solution of the entire present problem was not obtainable from that work, the present approach was philosophically the same.

### PROCESS DEFINITION AND THRESHOLD CROSSING AVERAGES

As noted in the main part of this report, the process,  $Y(t)$ , is assumed to be Gaussian and zero mean. With these assumptions the process may be described by its variance spectrum. Further, for present purposes, the zero<sup>th</sup> and second spectral moments are all that is required as a definition of the process. In particular,

$$\text{Zero}^{\text{th}} \text{ Spectral Moment} = m_0 = \sigma^2 = \text{the Variance of } Y(t)$$

$$\text{Second Spectral Moment} = m_2 = \sigma_v^2 = \text{the Variance of } \dot{Y}(t)$$

and  $\sigma$  is the standard deviation of  $Y(t)$  and  $\sigma_v$  is the standard deviation of the first time derivative of  $Y(t)$ .

Basic formulae for the average statistics of the time intervals between up (or down) crossings of a threshold have been commonly quoted for a long time (see for example, (Rice 1944, Ochi & Bolton 1973, Price & Bishop 1974, Lewis 1989). The most common of these is an estimate for a zero threshold, the *average zero-crossing period*,  $T_z$ ,

$$T_z = 2\pi \sqrt{\frac{m_0}{m_2}} = \frac{2\pi\sigma}{\sigma_v} \quad (2)$$

The expected number of up (or down) crossings of threshold  $a$  per unit time is given as

$$N_a = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \exp[-a^2 / 2m_0] = \frac{1}{T_z} \exp[-(a / \sigma)^2 / 2] \quad (3)$$

Inverting Eq. 3, the average time interval,  $T_a$ , between up-crossings of threshold  $a$  becomes

$$T_a = T_z \exp[(a/\sigma)^2 / 2]$$

#### AVERAGE DURATIONS ABOVE AND BELOW THE THRESHOLD

In a random and ergodic process (which is generally assumed) the probability that the process is greater than some value,  $a$ , is equal to the fraction of time that the process spends above  $a$ . Then for a Gaussian process with standard deviation  $\sigma$ ,

$$\text{Fraction of time that } Y(t) > a = \text{Prob}[Y(t) > a] = 1 - \Phi(a/\sigma)$$

where  $\Phi(r)$  is the standardized Normal probability integral:

$$\Phi(r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^r \exp[-t^2 / 2] dt$$

The average length of the time intervals during which the process is greater than  $a$  is denoted by  $\langle \tau_+ \rangle$  and estimated as:

$$\left\langle \tau_+ \right\rangle = \frac{\text{Fraction of time that } Y(t) > a}{\text{expected number of up crossings of } a \text{ per unit time}} = \frac{\Phi(a/\sigma)}{N_a}$$

and, using Eq. 3, the average length of the time intervals during which  $Y(t) > a$  becomes

$$\langle \tau_+ \rangle = T_z \exp[(a/\sigma)^2 / 2] [1 - \Phi(a/\sigma)] \quad (4)$$

Apart from notation, the estimate for  $\langle \tau_+ \rangle$ , Eq. 4, appears in much of the early literature, (Rice 1958) for example, and was used in the work by Graham et al (1991) that forms a model of sorts for the present work. The corresponding estimate for the average length of the time intervals during which the process is *less than*  $a$  is not given explicitly, but is easy to derive. Following the derivation above, average length of the time intervals during which the process is less than  $a$  is denoted by  $\langle \tau_- \rangle$  and estimated as

$$\left\langle \tau_- \right\rangle = \frac{\text{Fraction of time that } Y(t) < a}{\text{expected number of down crossings of } a \text{ per unit time}} = \frac{\Phi(a/\sigma)}{N_a}$$

where the statistical symmetry of Gaussian processes has been assumed in accordance with Eq. 3. Using Eq. 3 as before, the average length of the time intervals during which  $Y(t) < a$  becomes

$$\langle \tau_- \rangle = T_z \exp[(a/\sigma)^2/2] \Phi(a/\sigma) \quad (5)$$

Equation 5 has the potential sought as a criterion. It is very little more challenging to compute than the average up (or down) crossing period because a number of very accurate and fast numerical approximations to the Probability integral are available.

#### PROBABILITY DENSITIES OF THE INTERVALS: THE GENERAL CASE

Given an estimate of the average time intervals, it is of interest next to consider the possibility of estimates of the probability densities of the time intervals. Work on the various probability densities associated with the threshold crossing problem continued off and on for thirty years since the days of Rice (1958) and the nearly impenetrable work by McFadden (1956, 1958). Indeed, when written, the review paper by Blake and Lindsey (1973) had over one hundred references. The work may still be going on. The writer's experience with the general approaches in the literature ends at about the same era as the work by Graham (1991).

The first of the available approximations to the density of exceedance durations originates with the theory of Rice (1958), as refined by Kuznetsov, Stratanovich and Tikhoniv (Kuznetsov et al 1965, Tikhonov 1965). The theory and approximations are also described by Price and Bishop (1974). This exact theory deals with the conditional probability density for the duration of exceedance of an arbitrary threshold, with no pre-conditions about the nature of the process, or the statistical independence of the times of threshold crossing. The conditioning of the density is upon the value of the threshold, with a duration event defined as the time interval between successive up- and down-crossings of the threshold. This "exact" theory takes the form of an infinite functional series that was impossible to evaluate in the 1960's—and may still be. Practical use has involved approximations and assumptions of one sort or another.

The usual approach for arbitrary thresholds is due to Rice (1958) and Tikhonov (1965). This approximation involves taking only the first term of the infinite series of the exact theory as an approximation for arbitrary threshold levels but "small" durations.

Unfortunately, this approach yields a result which is not exactly that desired. In effect the approximate density involves the probabilities that, given a threshold up-crossing at time,  $t_0$ , there is a down-crossing of the threshold in the neighborhood of a later time,  $t$ , regardless of what happened between time  $t_0$  and  $t_0 + t$ . As a result, the approach appears to yield good approximations to the true density for short durations, but *does not integrate to unity!*. In fact the integral may not even be finite.

An essentially more modern treatment of the problem is due to Rainal (1987) and Mimaki (1978, 1981). In these works, the estimation of the conditional density was formulated in a different way, and by making the assumption that a given individual interval between threshold crossings is statistically independent of nearly all the possible sums of preceding intervals, the problem was reduced to the solution of a Volterra integral equation involving the numerical results from the approximation just described. On the whole, test numerical results were more satisfying than those of the first method. However, the method has quite disquieting faults—principally that the density can become negative for large durations!

Examples of the numerical results from both approaches appear in (Graham et al 1991). Noting that both these approaches yield results that violate the basic ideas of probability and probability densities, neither Graham or the writer had much faith in their practicality, and Graham looked elsewhere for approximations.

In the present case the requirement is for the densities of the below-threshold intervals rather than the above-threshold intervals. In view of the problems just recounted for the above-threshold intervals, it was doubted that the methods noted above could work any better for the below-threshold intervals.

## ASYMPTOTIC PROBABILITY DENSITIES OF THE INTERVALS

Graham (1991) pointed out that Rice (1958) had demonstrated that the probability density of exceedance intervals tends toward the Rayleigh distribution for very large thresholds; that is for rare threshold crossings, and that Vanmarcke (1975) had exploited this asymptotic result. Graham adopted the asymptotic result subject to a demonstration of validity relative to a simulation.

Taking the general form of the Rayleigh density for exceedance time intervals,  $\tau_+$ , to be

$$p_+(\tau_+) = \lambda \tau_+ \exp[-\lambda \tau_+^2 / 2]$$

and computing the expected value,

$$\langle \tau_+ \rangle = \int_0^\infty \tau_+^2 \lambda \exp[-\lambda \tau_+^2 / 2] d\tau_+ = \sqrt{\frac{\pi}{2\lambda}}$$

Solving for  $\lambda$  and using the result in the general form, and nondimensionalizing the above threshold intervals by the expected value, the expression for the asymptotic probability density of  $\tau_+ / \langle \tau_+ \rangle$  becomes:

$$p_+(\tau_+ / \langle \tau_+ \rangle) = \frac{\pi}{2} \frac{\tau_+}{\langle \tau_+ \rangle} \exp \left[ -\frac{\pi}{4} \left( \frac{\tau_+}{\langle \tau_+ \rangle} \right)^2 \right] \quad (6)$$

There was no clear indication in the modern literature what an asymptotic density for the below-threshold intervals might be, and it was necessary to re-read (Rice 1958) more carefully. It turns out that at the same time as he arrived at the asymptotic result for above-threshold intervals as the threshold approached positive infinity, Rice also arrived at an asymptotic result for above-threshold intervals as the threshold approached *negative infinity*. Because the process is statistically symmetrical, above-threshold intervals for negative thresholds are the same as below-threshold intervals for positive thresholds. Thus, though a little obscure, Rice also arrived at an asymptotic result for the below-threshold intervals as defined here. This asymptotic density is another simple one, the Exponential. The parameter of the Exponential density is the mean value. Nondimensionalizing the below-threshold intervals by the expected value, the expression for the asymptotic probability density of  $\tau_- / \langle \tau_- \rangle$  may be written

$$p_-(\tau_- / \langle \tau_- \rangle) = \exp \left[ -\frac{\tau_-}{\langle \tau_- \rangle} \right] \quad (7)$$

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## APPENDIX B. COMPARISON OF THEORETICAL AND SIMULATED RESULTS

### SIMULATION OF A SAMPLE PROCESS

It was important to find some "data" that could be used to confirm the estimating formulae for the average below-threshold durations and to see how reasonable the asymptotic densities were. As in (Graham et al 1991) a numerical simulation was resorted to.

It was thought reasonable to simulate realizations of a typical wave spectrum since, generally, the wave spectrum is the least narrow banded of all the seakeeping processes. Generally, the retrieval operation is carried out at relatively low ship speed, and it was not thought worth the complication to transform the spectrum to the encounter domain.

Arbitrarily, a Bretschneider spectral form with modal period of 14.1 seconds and significant height of 4 units was chosen. With this significant height choice, the elevation standard deviation,  $\sigma$ , is unity. The choice of modal period results in a value of zero crossing period,  $T_Z = 10$  seconds.

The numerical simulation method employed is often called "fast convolution". In this recipe a time series containing an approximation to a band-limited white Gaussian noise is first generated. The underlying Gaussian noise generator is reasonably accurate to values about five times  $\sigma$ . The complex FFT spectrum of the noise is then computed. A nonrealizable frequency domain filter corresponding to the desired variance spectrum is formed by simply square-rooting the desired variance spectrum, and this is applied to the FFT noise spectrum to produce a complex FFT spectrum of a realization of the process. The final step is an FFT inverse transform which produces a time series realization of the process.

The time series simulations of realizations of the wave spectrum selected were generated in "handy-sized" realizations of 8K points. The net result of each simulation was a time series of 8000 points which represented approximately one half hour of real time.

Fifty independent realizations of the process were simulated by entering the computer noise generator at widely separated points in its sequence. Considering all of

the samples, the pooled result is equivalent to an observation of the wave process defined by the 14.1 second modal period spectrum for about 25 hours.

As a check, the mean variance spectrum of the simulation was estimated as though the results were “data”. Because of the enormous sample, the spectral analysis involved 300 degrees of freedom per spectral estimate, and the resulting spectral estimates corresponded to the initial Bretschneider spectrum to well within the corresponding analysis confidence bounds of +15% to minus 12%. The sample variance was within 1.% of that specified — also within the analysis confidence bounds of approximately  $\pm 4\%$ .

Figure B.1 shows one of the 50 simulated wave realizations. The time scale is “real time” because the simulation code produced dimensional results, and continues down the eight frames of the figure.

A threshold crossing analysis was performed on each of the simulated realizations for each of the six nondimensional thresholds: 0.0,  $0.5\sigma$ ,  $1.0\sigma$ ,  $1.5\sigma$ ,  $2.0\sigma$  and  $2.5\sigma$ . Given an assumed value of the threshold, the analysis amounted to first finding the times of up and downcrossings throughout the realization. Time differences between each up and down-crossing are counted as an above-threshold duration,  $\tau_+$ , and the time difference between each down and upcrossing as a below-threshold duration,  $\tau_-$ . Threshold crossings at the ends of the realizations were ignored if they did not define an above and below threshold pair; that is, the same number of above and below intervals were derived from each simulated realization. Because this convention eliminates the effects of the relatively arbitrary starting and ending times of each realization, the sample above and below threshold intervals from the analysis of each of the fifty realizations were pooled to form one large sample for each assumed threshold value.

## COMPARISON OF MEAN VALUES FROM THEORY AND SIMULATION

The average value of the pooled above and below threshold intervals from the simulation were calculated and are compared with the corresponding theoretical results (Eqs. 4 and 5) in Fig. B.2. Both the theoretical and simulated means have been nondimensionalized by the zero crossing period,  $T_z$ , before plotting. The top frame pertains to the above-threshold average values and the lower to the below-frame intervals. For a zero threshold both averages amount to half the zero crossing period. The

above-threshold averages decrease to about 15% of  $T_Z$  at a threshold of  $2.5\sigma$  while the below-threshold averages increase to about 20 times  $T_Z$  at the  $2.5\sigma$  threshold. The theoretical estimates, Eqs. 4 and 5 appear to agree quite well with the simulated averages.

#### TESTS OF FIT TO THE ASYMPTOTIC DISTRIBUTIONS

Though previous work (Graham et al 1991, Vanmarcke 1975) has suggested that the asymptotic distribution for the above-threshold time intervals (the Rayleigh) is reasonable for thresholds comparable to those used here, there was no encouragement about the reasonableness of the Exponential for the below-threshold intervals. Accordingly, the simulated samples of time intervals were subjected to a number of hypothesis tests to see if any of the samples could be accepted in an objective way as originating from the asymptotic distributions.

Figure B.3 summarize the results of these various tests on the samples of above and below threshold intervals for the  $2.5\sigma$  threshold. The plots are rotated to fit two to the page. The left side of the rotated page contains the results for the above-threshold intervals and the right side the results for the below-threshold intervals.

Each of the charts indicates the adequacy of fit of a sample to the assumed probability distribution (Rayleigh for above-threshold and Exponential for below-threshold intervals.) The fitted distributions and the sample frequency distributions are plotted in a "probability paper" form. The values of the fitted distribution parameter are shown at the lower right corner of each plotting field. The plot abscissa normally pertains to dimensional values. In the present case the samples were nondimensionalized before the tests began by the sample mean, and the abscissa is nondimensional in the present case. The nominal ordinate (on the left) is a scale of probability that the variate is less than the value indicated. The actual ordinate on a probability paper is a linear scale of "reduced variate", shown to the right. The reduced variate is normally a non-dimensional form of the distribution argument. The transformation between the reduced variate and the probability scales is controlled by the formula for the distribution and the fitted values of the parameters. The effect is that the theoretical fitted distribution vs. dimensional variate will plot as a straight line on a probability paper constructed for it.

In each plot, points on the corresponding sample distribution are shown as a series of square symbols. Sample distributions are basically obtained from a tally of the input

data into class intervals. In the present methods, the *number* of class intervals,  $M_c$ , is determined according to sample size by an algorithm which approximates the recommendations in (Bendat & Piersol 1971) for numbers of class intervals thought to optimize the Chi-squared test for goodness of fit at the 5% level of significance. The actual boundaries of the  $M_c$  class intervals are determined from the theoretical distribution in such a way so that each has a probability  $1/M_c$  of containing a single random sample from the ideal population. The effect is that if the data sample is compatible with the fitted distribution there should be about the same number of data points tallied into each class interval. The plotted points in the charts represent the upper boundary of the first  $M_c - 1$  class intervals. The upper boundary of the  $M_c^{th}$  interval represents a sample frequency of 1.0 and cannot be plotted.

The extremes of the ( $n$  point) data sample are plotted at probabilities of  $n/(n + 1)$  and  $1/(n + 1)$  in accordance with the typical procedure (Gumbel 1958).

For probability levels between 0.1 and 0.9 "90% Control curves" after the recommendations of Gumbel (1958) are included on the plots. These control curves are extended from probability levels of 0.1 and 0.9 to the plotting positions of the extremes in the sample by drawing straight lines to the ends of the 90% confidence bands on the extremes. The general idea of the control curves is to aid in judging the adequacy of the agreement of the sample with the fitted distribution. If the trend of the sample distribution does not agree with the fitted distribution, *and* more than about 10% of the plotted sample distribution points are outside the control curves, there is reason to reject the idea that the sample might have been drawn from a population having the fitted distribution.

The software which develops the numerical data to make the plots also applies two common goodness of fit tests. The first is the Kolmogorov-Smirnov test which is in effect applied to all individual pieces of the sample. The second is the Chi-squared test which is applied to the sample distribution shown in the plots. The results of both tests are reported in an unconventional way in two lines just above the plotting field. What is reported is the level of significance which would have had to be assumed in advance in order that the tests pass. The typical level of significance assumed in advance for such hypothesis tests is 5%.

Thus, if the level reported is about 5% or greater, the hypothesis that the sample came from the fitted distribution would normally be accepted. The higher the number the more firmly the test is passed and the more likely the sample is from the fitted distribution. On the other hand, a level of significance near zero indicates a highly significant *failure* of the test, and the hypothesis that the sample came from the fitted distribution can hardly be accepted on that basis of that test.

The results of the tests that are summarized for the  $2.5\sigma$  threshold in Fig. B.3 are quite gratifying. The numerical goodness of fit tests are passed very handily and it appears that all of the sample frequency estimates fall within the control curves except for the maximum of the above-threshold intervals. The result is interpreted to mean that both the asymptotic distributions can be accepted as very solid approximations for thresholds as low as  $2.5\sigma$ .

Fig. B.4 shows the results of the application of the same testing routines on the interval samples obtained for the  $2.0\sigma$  threshold. There is an obvious degradation in the fit. The Rayleigh fit to the above-threshold intervals would probably be considered marginal since some of the tests are passed. The Exponential fit to the below-threshold intervals is less good. Probably because of an apparent anomaly for low values of the interval, the numerical goodness of fit tests are significantly failed. However, the high value end of the below-threshold interval sample frequency curve conforms well to the control curves and this fit might also be classified as marginal overall.

The goodness of fit tests were applied to the rest of the interval samples for lower thresholds. As has to be expected, the fits steadily deteriorated as the threshold was reduced from  $2.0\sigma$ .

#### COMPARISONS OF THE ASYMPTOTIC AND SIMULATED DENSITIES

Following (Graham et al 1991) the next step involved direct comparisons of results of the simulation with the corresponding asymptotic probability densities. As in the preceding tests, each member of the samples of above and below threshold intervals was nondimensionalized by the respective mean value. In order to derive an estimate of the mean probability density over some small interval of duration, a uniform duration class interval,  $\Delta(\tau_+ / \langle \tau_+ \rangle)$  or,  $\Delta(\tau_- / \langle \tau_- \rangle)$ , was first assumed and the number of nondimensional simulated above or below threshold intervals falling into each was

tallied. The resulting tallies were divided by the total number of intervals in the sample to form an estimate of the probabilities,  $[p_+(\tau_+ / \langle \tau_+ \rangle) \Delta(\tau_+ / \langle \tau_+ \rangle)]$ , and  $[p_-(\tau_- / \langle \tau_- \rangle) \Delta(\tau_- / \langle \tau_- \rangle)]$ , and then these results were divided by the respective interval to form an estimate of the mean probability density over the interval. For graphical purposes the estimates were associated with the mid-point of the class interval.

The results of this procedure for the simulated intervals for the  $2.5\sigma$  threshold are shown in Fig. B.5. The upper frame gives the results from the simulated above-threshold intervals as compared to the asymptotic Rayleigh density, Eq. 6. Similarly, the lower frame gives the results from the simulated below-threshold intervals as compared to the asymptotic Exponential density, Eq. 7. As would be expected from the goodness of fit tests just described, the comparison of results from the simulation with the asymptotic theory is very favorable.

The results of the same procedure applied to the samples of simulated intervals for the other thresholds are given in Figs. B.6 through B.9.

Subjectively, the correspondence between theory and the simulation for the  $2.0\sigma$  and the  $1.5\sigma$  thresholds, Figs. B.6 and B.7 is as reasonable as that shown in Fig. B.5 for the case where there is little doubt that the asymptotic theory holds well. It appears that for reasonable engineering purposes the Exponential model may be employed for the below-threshold intervals of most interest here.

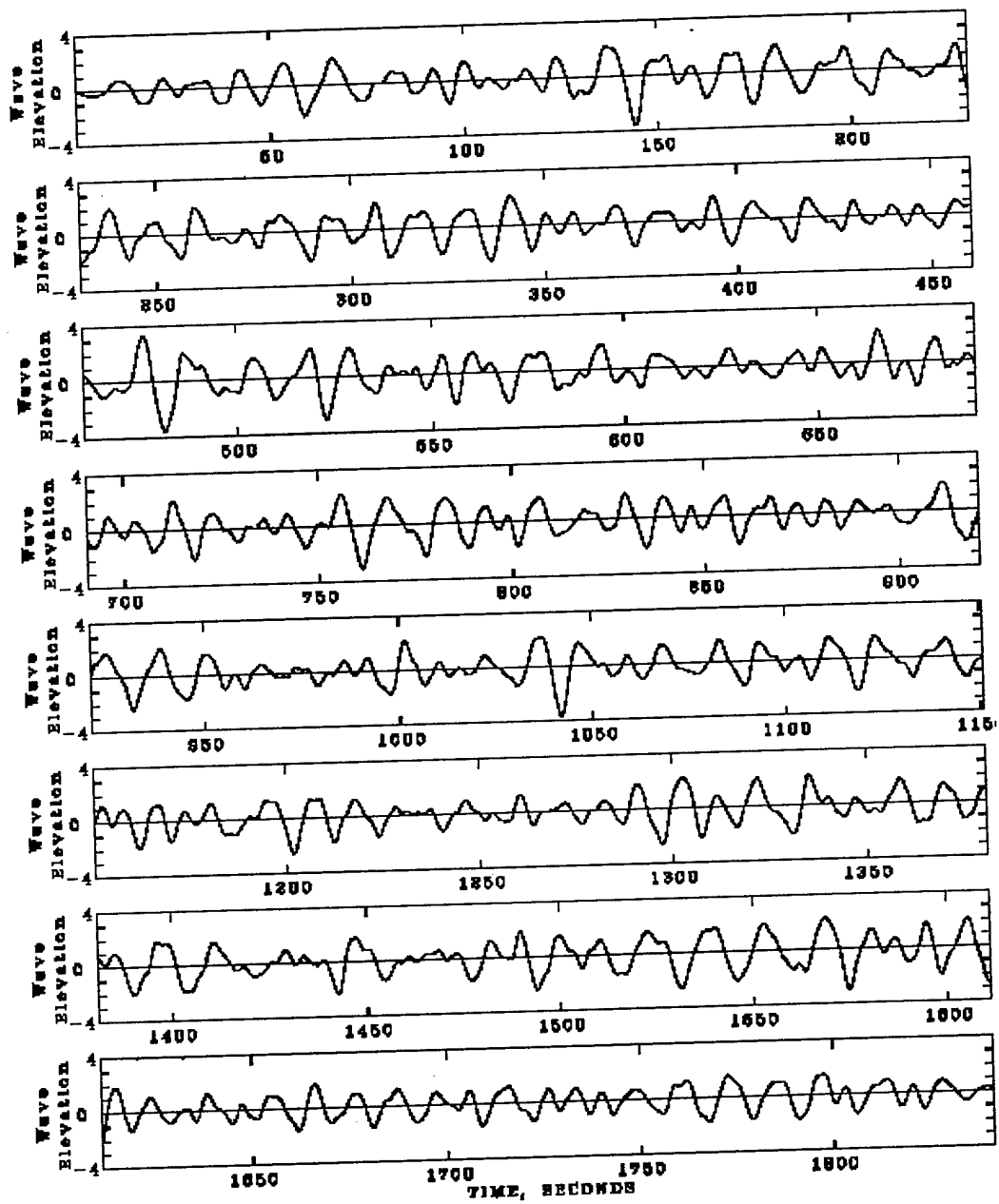
The same might be said for the above-threshold case for the  $1.0\sigma$  case shown in Fig. B.8, and this would be in accordance with Graham's (1991) conclusion. However, the simulated densities for the below-threshold intervals in Fig. B.8 deviate strongly from the Exponential for short intervals.

Finally, all results for the  $0.5\sigma$  threshold simulations, Fig. B.9 deviate more or less strongly from the asymptotics. The results from the simulations for the above-threshold intervals are at least qualitatively similar to the theory. However, those from the simulations for the below-threshold intervals are *not* qualitatively similar, and it has to be concluded that the asymptotic theory is way off the mark at this threshold level.

## SUMMARY

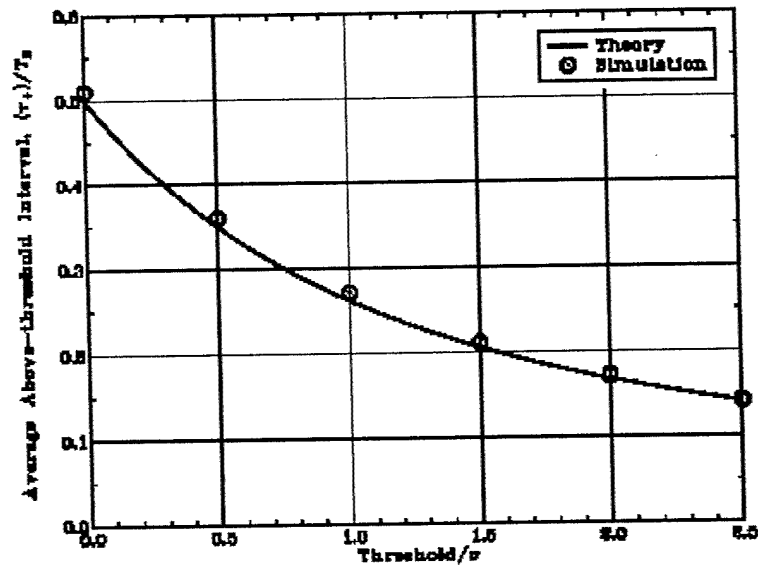
On the basis of comparisons of the results of simulations with the theory of Appendix A, it appears that

- The estimators for the average length of above and below threshold intervals, Eqs. 4 and 5 appear to be quite accurate for any threshold level.
- Objective goodness of fit tests suggest that the asymptotic probability densities for above and below threshold intervals are likely to hold for any purpose for thresholds of  $2.5\sigma$  and up. Essentially, at this threshold level the threshold crossings appear sufficiently rare.
- The conclusion of Graham (1991) is more or less confirmed that for many engineering purposes the asymptotic Rayleigh density can be used for above-threshold intervals for thresholds greater than  $1.0\sigma$ .
- Similarly, it appears that for many engineering purposes the asymptotic Exponential density can be used for below-threshold intervals for thresholds greater than  $1.5\sigma$ .
- For threshold levels of  $1.0\sigma$  and lower it appears that the probability density of the actual below-threshold intervals is qualitatively different than the Exponential approximation, and accordingly, no reasonable approximation to the probability density is available for this case.

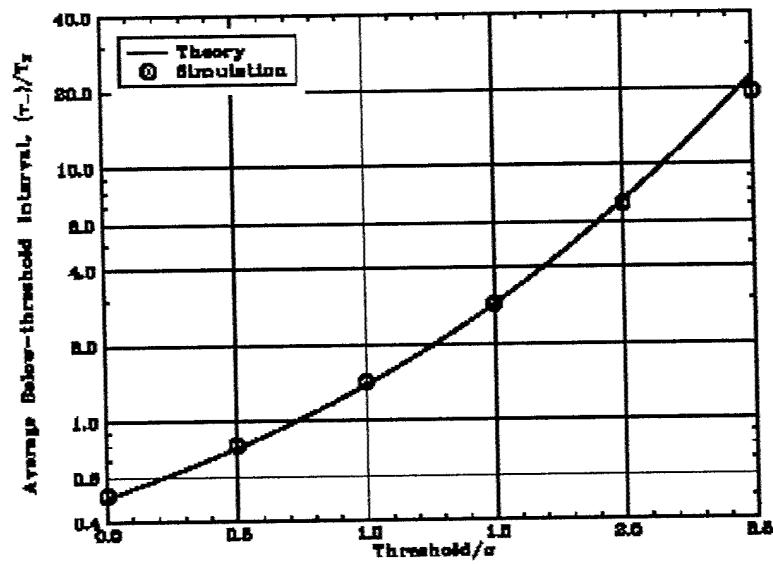


SAMPLE 1 : BREITSCHEIDER SPECTRUM,  $T_m = 14.1$  SEC,  $H_{1/3} = 4$ .

Figure B.1: Sample of the Simulated Wave Process



a) Above-threshold intervals.



b) Below-threshold intervals.

Figure B.2: Comparison of Simulated Average Intervals with Theory

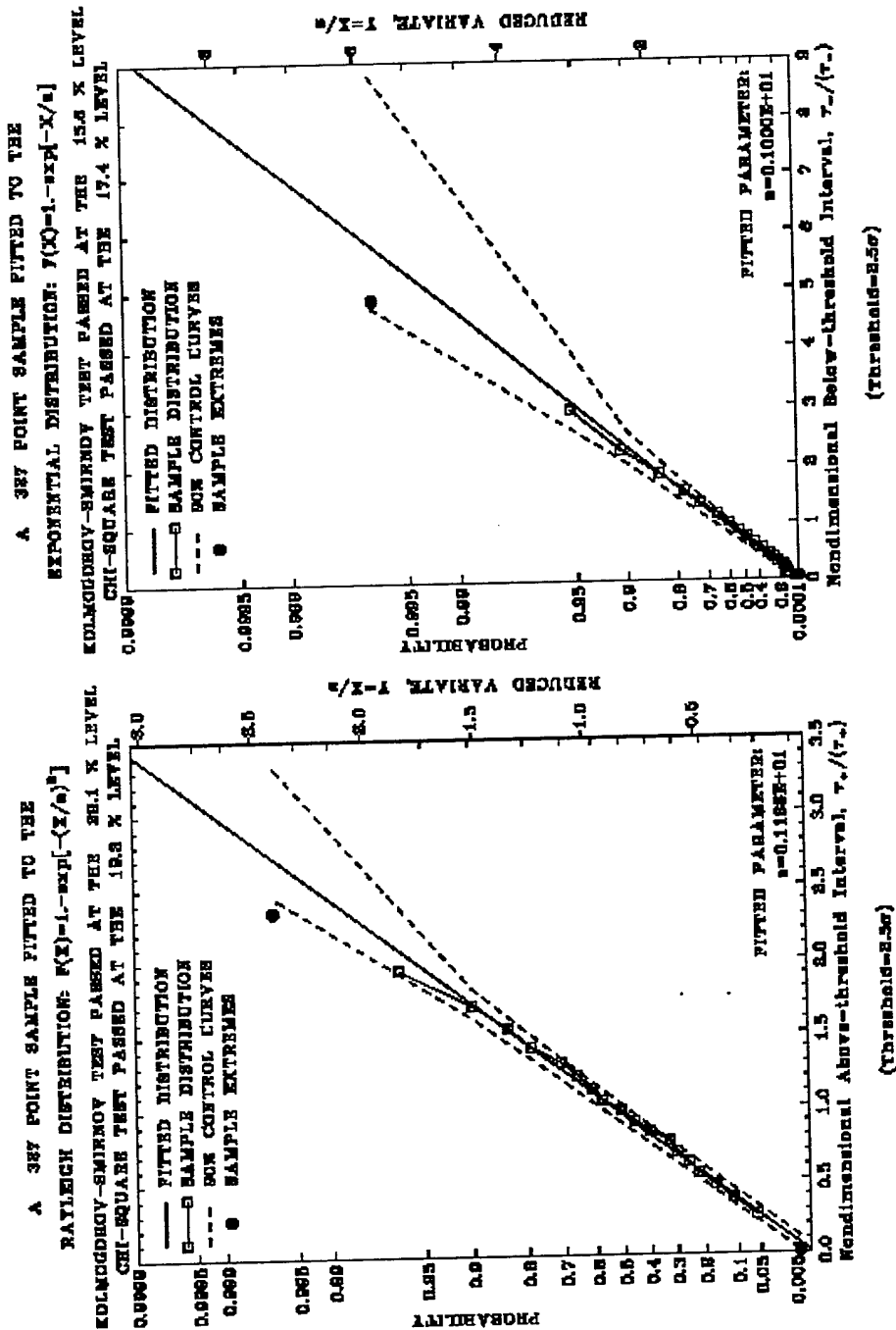


Figure B.3: Tests of Fit of Simulated Interval Samples to the Asymptotic Distributions for Threshold of 2.5 $\sigma$

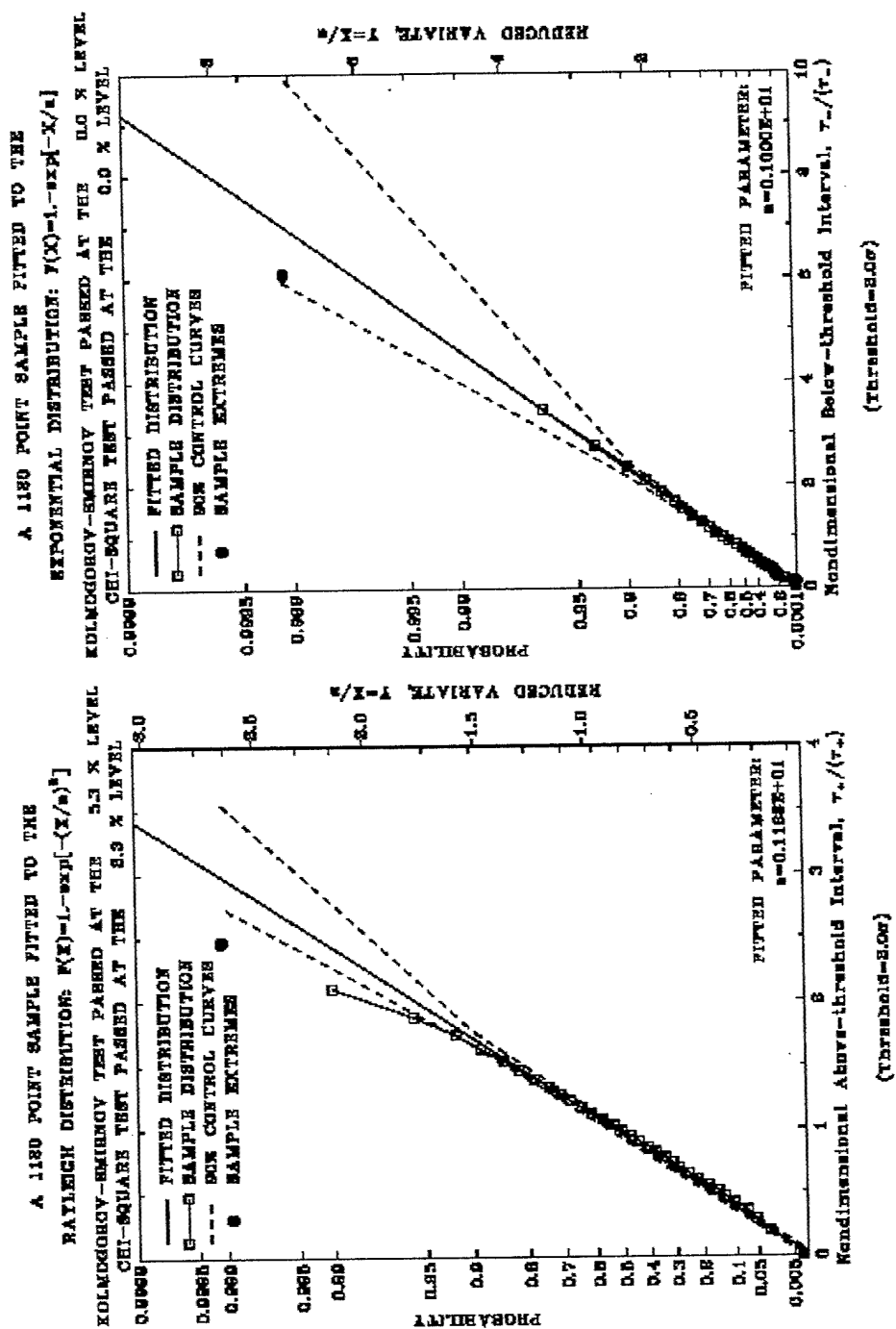


Figure B.4: Tests of Fit of Simulated Interval Samples to the Asymptotic Distributions for Threshold of  $2.0\sigma$

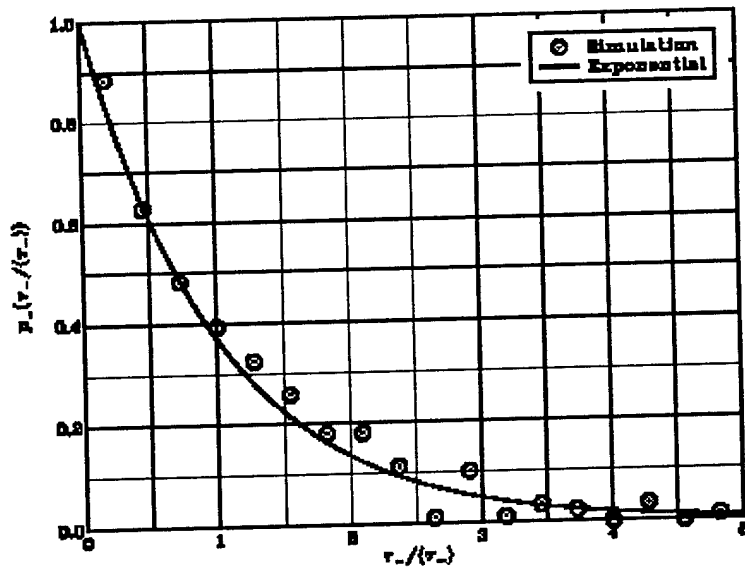
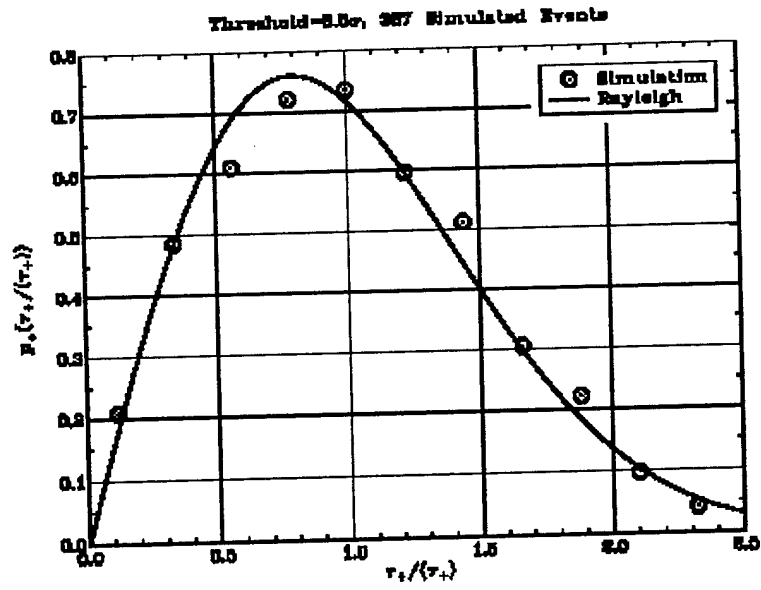


Figure B.5: Comparison of Theoretical and Simulated Densities, Threshold  $2.5\sigma$

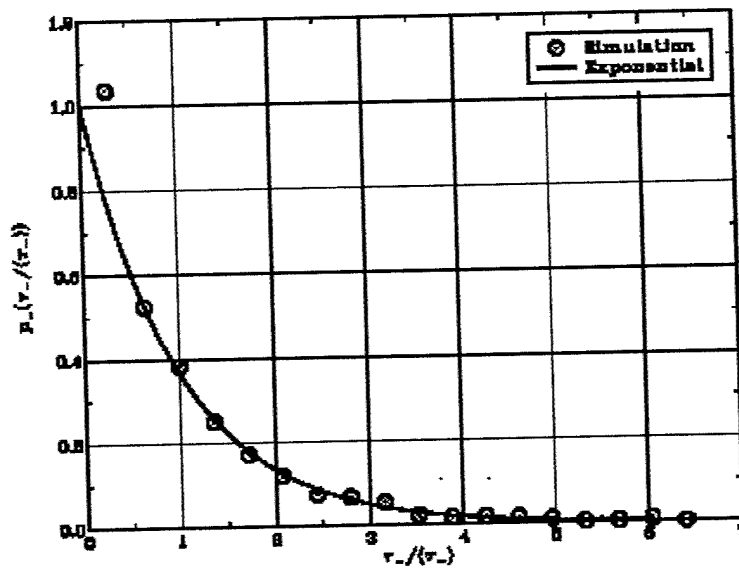
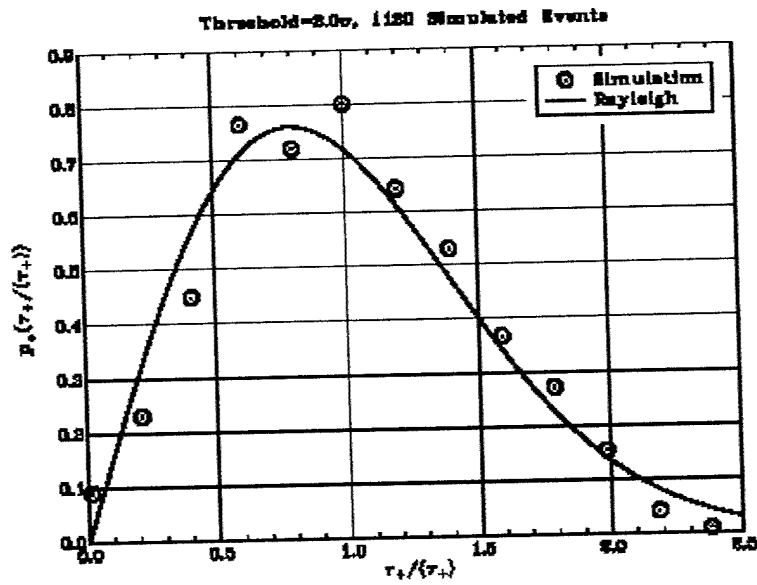
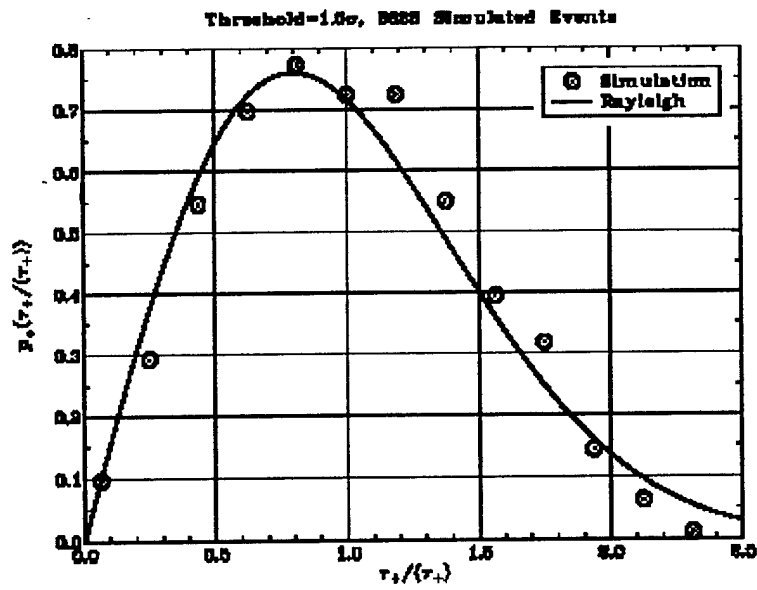
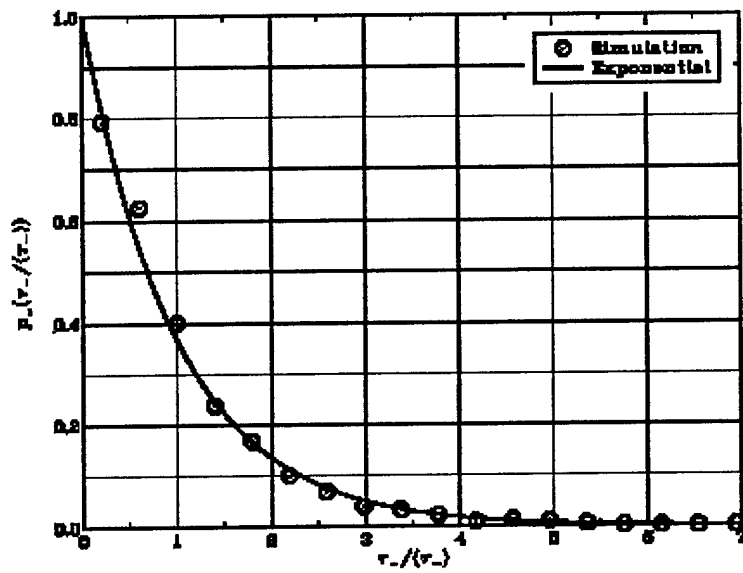


Figure B.6: Comparison of Theoretical and Simulated Densities, Threshold  $2.0\sigma$



a) Above-threshold intervals.



b) Below-threshold intervals.

Figure B.7: Comparison of Theoretical and Simulated Densities, Threshold 1.5 $\sigma$

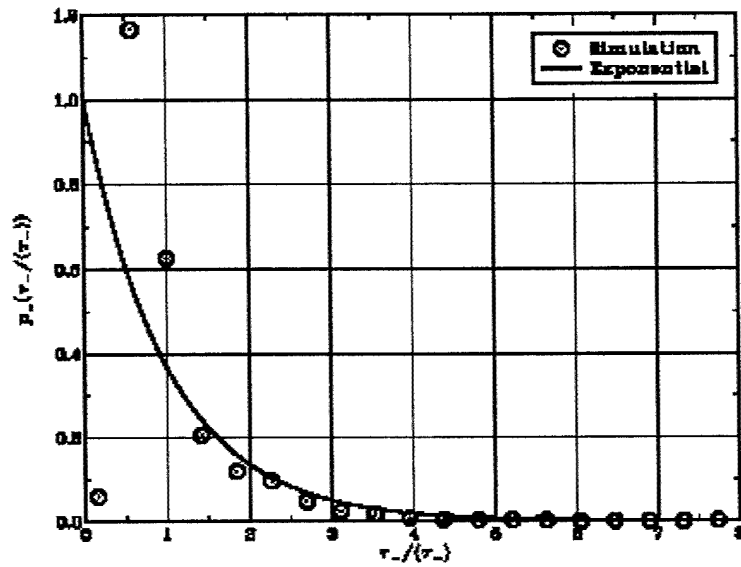
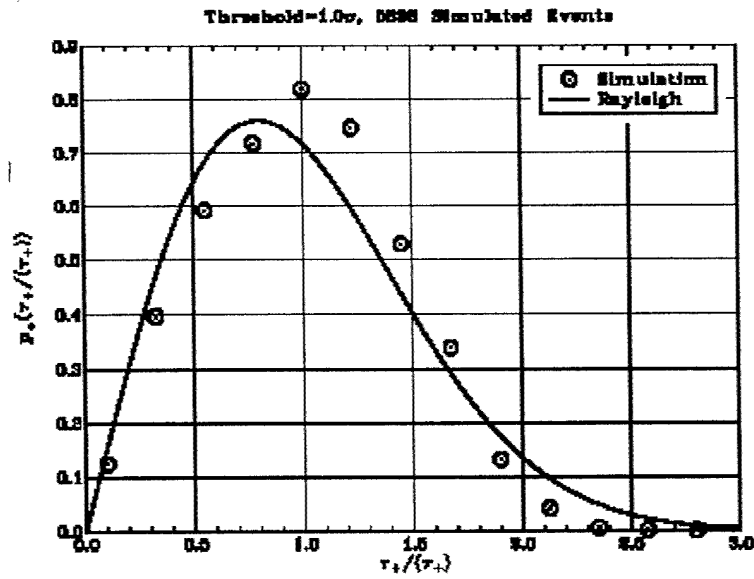


Figure B.8: Comparison of Theoretical and Simulated Densities, Threshold 1.0 $\sigma$

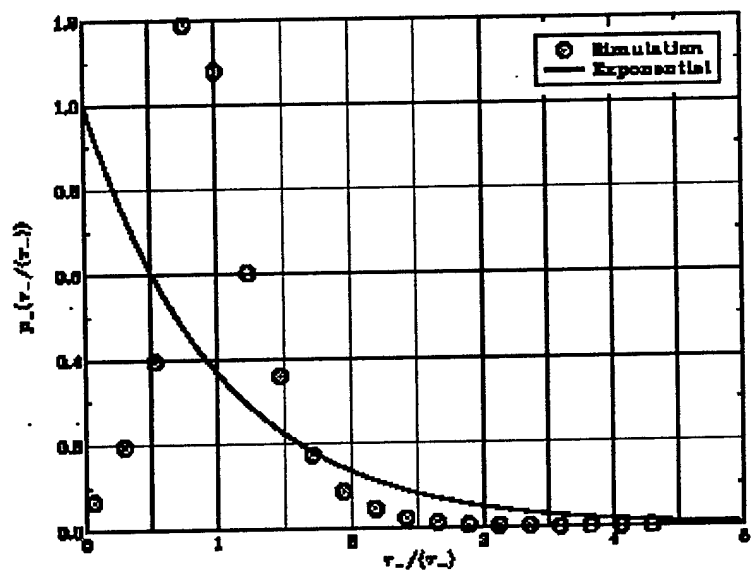
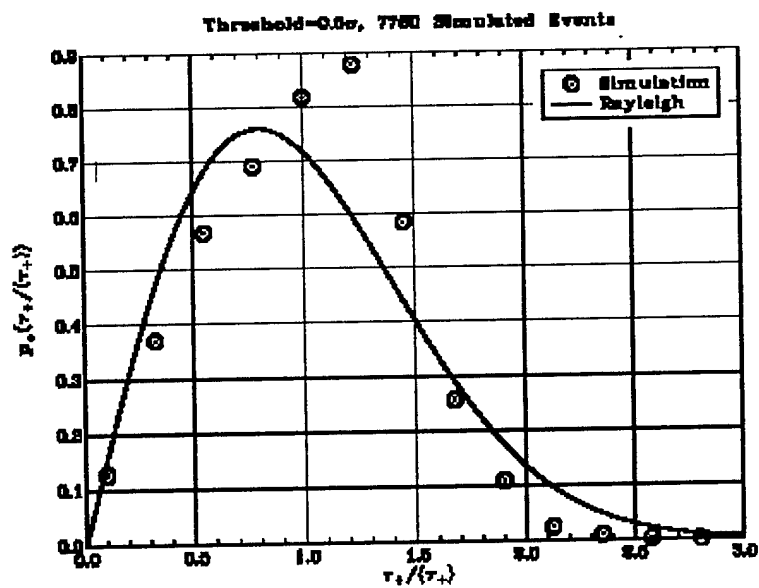


Figure B.9: Comparison of Theoretical and Simulated Densities, Threshold  $0.5\sigma$

## REFERENCES

- Bendat, J. S. and Piersol, A. G. 1971 *RANDOM DATA: Analysis and Measurement Procedures*. Wiley-Interscience, New York.
- Blake, I. F. and Lindsey, W. C. 1973 Level crossing problems for random processes. *IEEE Transactions on Information Theory*, **IT-19**, 3, May.
- Graham, R., Baitis, A. E. and Meyers, W. G. 1991 A frequency-domain method for estimating the incidence and severity of sliding. David Taylor Research Center Report DTRC/SHD-1361-01. Aug.
- Gumbel, E. J. 1958 *Statistics of Extremes*. Columbia University Press, New York
- Kuznetsov, P. I., Stratonovich, R. L. and Tikhonov, V. I. 1965 On the durations of excursions of random functions. In: *Non-Linear Transformations of Stochastic Processes*, P. I. Kuznetsov et al , Eds. Pergamon Press, Oxford.
- Lewis, E. V., Ed. 1989 *Principles of Naval Architecture*. Vol. III, Motions in Waves and Controllability. The Society of Naval Architects and Marine Engineers, Jersey City, NJ.
- McFadden, J. 1956 The axis-crossing intervals of random functions. *IRE Transactions on Information Theory*, **IT-2**, 146–150.
- McFadden, J. 1958 The axis-crossing intervals of random functions—II. *IRE Transactions on Information Theory*, **IT-4**, 14–24.
- Mimaki, T. and Munakata, T. 1978 Experimental results on the level-crossing intervals of gaussian processes. *IEEE Transactions on Information Theory*, **IT-24**, 4, July.
- Mimaki, T., Tanabe, M. and Wolf, D. 1981 The multipeak property of the distribution densities of the level-crossing intervals of a gaussian random process. *IEEE Transactions on Information Theory*, **IT-27**, 4, July.
- Ochi, M. K. and Bolton, W. E. 1973 Statistics for prediction of ship performance in a seaway. *International Shipbuilding Progress*, **20 & 21**.
- Price, W. G. and Bishop, R. E. D. 1974 *Probabilistic Theory of Ship Dynamics*. Chapman and Hall, London.

- Rainal, A. J. 1987 First and second passage times of Rayleigh processes. *IEEE Transactions on Information Theory*, **IT-33**, 3, May.
- Rice, S. O. 1944 Mathematical analysis of random noise. *Bell System Technical Journal*, **23**.
- Rice, S. O. 1958 Distribution of the duration of fades in radio transmission. *Bell System Technical Journal*, **37**, 3.
- Tikhonov, V. I. 1965 The distribution of the duration of excursions of normal fluctuations. In: *Non-Linear Transformations of Stochastic Processes*, P. I. Kuznetsov et al , Eds. Pergamon Press, Oxford.
- Vanmarcke, E. H. 1975 On the distribution of the first-passage time for normal stationary random processes. *Journal of Applied Mechanics*, **42**, 1, March.

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